

Revisiting Riccioli's free fall calculations

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In 1651 Giovanni Riccioli reported the earliest accurate measurements the acceleration due to gravity, g , from pendulum-timed free fall experiments. The use of Huygens' pendulum formula (published 1673) allows one to deduce the pendulum length from this data, free from assumptions about the conversion to modern units, and independent of the actual value of g . When this length is compared to the reported pendulum length, a 15% systematic error is revealed. This could perhaps be attributed to the difficulty Riccioli faced in subdividing the contemporary unit of length (the Roman foot) to the requisite millimeter accuracy.

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In 1651 Giovanni Riccioli published measurements of the acceleration due to gravity from pendulum-timed free fall experiments, in his *Almagestum Novum* (a treatise on mathematics and astronomy, updating Ptolemy's 2nd century *Almagest*). Riccioli confirmed Galileo's observation reported twenty years previously (in 1632) that the distance z fallen in a time t satisfies $z \propto t^2$. Riccioli's measurements were the most accurate to that date of the constant of proportionality, which in modern terminology is $g/2$. A clear-cut comparison with the current accepted value $g \approx 9.81 \text{ m s}^{-2}$ is frustrated by uncertainty over the conversion of Riccioli's unit of length, the Roman foot, to modern units. All this has been elegantly explained in an article in Physics Today by Graney [1], who also provides a modern translation of the relevant part of the *Almagestum Novum* [2]. Graney's article was the inspiration for this short note, which extends the present author's recently published Letter in Physics Today [3].

Riccioli's actual free fall data is shown in Table I. The second column in the Table is the free fall time, measured in pendulum half periods (approximately 1/6 second). The fifth column in the Table is the free fall distance, measured in Roman Feet. Unbeknownst to Riccioli at the time, but known to us thanks to Christiaan Huygens, the half period of a pendulum is $T = \pi\sqrt{L/g}$ where L is the length of the pendulum. This can be combined with the free fall law, $z = gt^2/2$, to get

$$z = \frac{\pi^2 L}{2} \times \left(\frac{t}{T}\right)^2. \quad (1)$$

So a plot of the free fall distance against the square of the free fall time measured in pendulum half periods should be a straight line through the origin with a slope $\pi^2 L/2$. Hence we can work out the length of the pendulum, at least in Roman Feet. Riccioli's data plotted in this way is shown in Fig. 1: it is a very good fit to a straight line. The best fit line (through the origin) has slope $\pi^2 L/2 = 0.412(2)$ where the figure in brackets is the reported least squares fitting error in the final digit. Hence $L = 0.0834(4)$ Roman feet, or 1.002(5) Roman inches.

Odo experi mento rum	Vibrationes Simplices Pen- duli altitudi- nem viciatim 1, 1/2, 3/4, 1/3, 1/4, 1/5, 1/6, 1/8, 1/10, 1/12, 1/15, 1/20, 1/24, 1/30, 1/36, 1/40, 1/48, 1/60, 1/72, 1/80, 1/90, 1/100, 1/120, 1/144, 1/160, 1/180, 1/200, 1/225, 1/240, 1/270, 1/300, 1/324, 1/360, 1/400, 1/450, 1/480, 1/500, 1/540, 1/600, 1/640, 1/675, 1/720, 1/750, 1/800, 1/810, 1/864, 1/900, 1/960, 1/1000, 1/1080, 1/1125, 1/1200, 1/1250, 1/1296, 1/1350, 1/1440, 1/1500, 1/1560, 1/1600, 1/1620, 1/1680, 1/1728, 1/1800, 1/1875, 1/1920, 1/1944, 1/2000, 1/2025, 1/2070, 1/2100, 1/2160, 1/2250, 1/2304, 1/2340, 1/2400, 1/2430, 1/2450, 1/2500, 1/2520, 1/2560, 1/2600, 1/2640, 1/2700, 1/2736, 1/2760, 1/2800, 1/2850, 1/2880, 1/2916, 1/2940, 1/2970, 1/3000, 1/3024, 1/3060, 1/3100, 1/3120, 1/3150, 1/3168, 1/3200, 1/3240, 1/3270, 1/3300, 1/3360, 1/3400, 1/3450, 1/3500, 1/3520, 1/3560, 1/3600, 1/3640, 1/3675, 1/3700, 1/3744, 1/3780, 1/3800, 1/3840, 1/3870, 1/3900, 1/3960, 1/4000, 1/4050, 1/4080, 1/4100, 1/4140, 1/4176, 1/4200, 1/4230, 1/4250, 1/4280, 1/4320, 1/4350, 1/4375, 1/4400, 1/4440, 1/4470, 1/4500, 1/4536, 1/4560, 1/4600, 1/4640, 1/4675, 1/4700, 1/4740, 1/4770, 1/4800, 1/4830, 1/4860, 1/4896, 1/4920, 1/4950, 1/4980, 1/5000, 1/5040, 1/5070, 1/5100, 1/5136, 1/5160, 1/5190, 1/5200, 1/5220, 1/5250, 1/5280, 1/5310, 1/5340, 1/5375, 1/5400, 1/5430, 1/5460, 1/5490, 1/5520, 1/5550, 1/5580, 1/5616, 1/5640, 1/5670, 1/5700, 1/5730, 1/5760, 1/5790, 1/5800, 1/5820, 1/5850, 1/5880, 1/5910, 1/5940, 1/5970, 1/6000, 1/6030, 1/6060, 1/6090, 1/6120, 1/6150, 1/6180, 1/6216, 1/6240, 1/6270, 1/6300, 1/6330, 1/6360, 1/6390, 1/6400, 1/6420, 1/6450, 1/6480, 1/6510, 1/6540, 1/6575, 1/6600, 1/6630, 1/6660, 1/6690, 1/6720, 1/6750, 1/6780, 1/6816, 1/6840, 1/6870, 1/6900, 1/6930, 1/6960, 1/6990, 1/7000, 1/7020, 1/7050, 1/7080, 1/7110, 1/7140, 1/7175, 1/7200, 1/7230, 1/7260, 1/7290, 1/7320, 1/7350, 1/7380, 1/7416, 1/7440, 1/7470, 1/7500, 1/7530, 1/7560, 1/7590, 1/7600, 1/7620, 1/7650, 1/7680, 1/7710, 1/7740, 1/7775, 1/7800, 1/7830, 1/7860, 1/7890, 1/7920, 1/7950, 1/7980, 1/8000, 1/8040, 1/8070, 1/8100, 1/8136, 1/8160, 1/8190, 1/8200, 1/8220, 1/8250, 1/8280, 1/8310, 1/8340, 1/8375, 1/8400, 1/8430, 1/8460, 1/8490, 1/8520, 1/8550, 1/8580, 1/8616, 1/8640, 1/8670, 1/8700, 1/8730, 1/8760, 1/8790, 1/8800, 1/8820, 1/8850, 1/8880, 1/8910, 1/8940, 1/8970, 1/9000, 1/9030, 1/9060, 1/9090, 1/9120, 1/9150, 1/9180, 1/9216, 1/9240, 1/9270, 1/9300, 1/9330, 1/9360, 1/9390, 1/9400, 1/9420, 1/9450, 1/9480, 1/9510, 1/9540, 1/9575, 1/9600, 1/9630, 1/9660, 1/9690, 1/9720, 1/9750, 1/9780, 1/9816, 1/9840, 1/9870, 1/9900, 1/9930, 1/9960, 1/9990, 1/10000	Tempus primi Mobilis respon- dens Vibratio- nibus.	Numeri Quadrati Vibratio- num.	Spacia cōfecta à Globo argilla- cei Viciatim 8. in fine temporis.	Spacia fecerim cōfecta fingulis temporibus.	Proportio Incre- menti Velocita- tis Grauium in Aëre nōstrate.
	Vibr. Simpl.	Secūda Tertia	Quadrata	Pedes Romani	Pedes Romani	Numeri minimi
I.	1 10 15 20 25	0" 1 40" 2 30" 3 20" 4 10"	1 100 225 400 625	10 40 90 160 250	10 30 50 70 90	1 3 5 7 9
II.	6 12 18 24 30	1 0 2 0 3 0 4 0 5 0	36 144 324 576 810	15 60 135 240 360	15 45 75 105 135	1 3 5 7 9
III.	6 1/2 13 1/2 20 1/2 27 1/2 34 1/2	1 1/2 2 1/2 3 1/2 4 1/2 5 1/2	42 169 381 676 1000	18 72 162 288 450	18 54 90 126 162	1 3 5 7 9

TABLE I. Riccioli's original data from the *Almagestum Novum* (p387). The table was taken from Google Books.

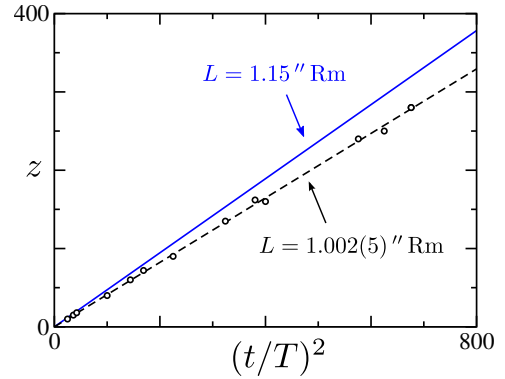


FIG. 1. A plot of the free fall distance as a function of the square of the free fall time measured in pendulum half periods should obey Eq. (1). The circles are Riccioli's data, with a best-fit line (dashed). The solid line (blue) is the prediction using Riccioli's reported pendulum length.

This can be compared with Riccioli's report of the pendulum length which "measured to the center of the little bob is one and fifteen hundredths of the twelfth part of an old Roman foot" [2]. So Riccioli has $L = 1.15$ Roman inches (with an inferred accuracy of 5 parts in 100). The prediction from this is shown as the solid (blue) line in Fig. 1. Clearly, a systematic error has crept in somewhere. Moreover, the discrepancy is independent of the

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conversion to modern units, and independent of the actual value of g , which cancels in Eq. (1).

The data is consistent with the pendulum being 15% shorter than Riccioli claimed, or equivalently the oscillation frequency being 7% too rapid (recall $T \sim L^{1/2}$). However the discrepancy is in the wrong direction to be explained either by the fact (known to Riccioli by that time) that large amplitude pendulum swings are no longer isochronous (otherwise the pendulum would be slower than expected) or that it should properly be regarded as a compound pendulum whose centre of oscillation lies below the centre of mass (again it would be slower than expected). Unless there is a typographical error, the most plausible explanation seems to lie with Riccioli's measurement of the pendulum length, perhaps in the difficulty in subdividing a Roman foot into 5/100 parts of Roman inches (*i. e.* 240 subdivisions). This is an accuracy of about 1 mm in modern units so a systematic error seems perfectly excusable.

Let me end with a philosophical digression. What entitles us to conclude there is a systematic error? The answer is that it doesn't agree with Huygens' theory for the period of a pendulum. But surely this is exactly backwards? Experiment is supposed to confirm theory, not theory disprove experiment! Actually in practised science this is much more common than might be expected, and has even been elevated to a principle. The precise quote, attributed to Arthur Eddington, is :

It is also a good rule not to put over much

confidence in the observational results that are put forward until they are confirmed by theory.

Galileo had something to say on systematic errors [4], discussing deviations from his observation that all objects fall at the same rate :

But, Simplicio, I trust you will not follow the example of many others who divert the discussion from its main intent and fasten upon some statement of mine which lacks a hairsbreadth of the truth and, under this hair, hide the fault of another which is as big as a ship's cable.

Physics is an experimental science and the nitty-gritty is largely about the control and elimination of sources of systematic error. Knowing when to stop, and awareness that "perfect is the enemy of good" [5], is I guess the mark of a good experimentalist.

For context, here's a brief chronology of related works published in the 17th century :

- 1632 – Galileo – *Dialogue*,
- 1651 – Riccioli – *Almagestum Novum*,
- 1673 – Huygens – *Horologium Oscillatorium*,
- 1687 – Newton – *Principia*.

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- [1] C. M. Graney, Physics Today **65** (9), 36 (September 2012); <http://dx.doi.org/10.1063/PT.3.1716>.
 [2] C. M. Graney, <http://arxiv.org/abs/1204.3267>.
 [3] P. B. Warren, Physics Today **66** (3), 8 (March 2013);

- <http://dx.doi.org/10.1063/PT.3.1896>.
 [4] *Dialogue Concerning the Two Chief World Systems*, Eng. trans. Henry Crew and Alfonso de Salvio (1638).
 [5] From Voltaire, *La Bégueule* (1772).